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评定直线度误差的最小二乘法与 最小包容区域法精度之比较

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摘要:介绍了直线度误差评定的最小二乘法和最小包容区域法的算法模型与实现方法。在三坐标测量机上对八种不同被测直线进行了采样点坐标数据提取,分别用最小二乘法和最小包容区域法的基于搜索逼近-逐次旋转逼近法进行了给定平面内直线度误差的评定。结果表明:最小二乘法的评定结果与最小包容区域法的基于搜索逼近-逐次旋转逼近法的评定结果完全一致,即直线度误差的最小二乘法评定结果符合最小条件。

关键词:直线度误差;评定;最小二乘法;最小包容区域法

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Comparison of evaluating precision of straightness error between least square method and least envelope zone method

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Abstract: The algorithm model and the realization method of evaluating straightness error in least squares method and least envelope zone method were introduced in detail. The experimental data of eight different kinds of straights were obtained on Coordinate Measuring Machine(CMM). Then the measured data were evaluated by least square method and least envelope zone method based on search approximation (namely cut and try method in revolutionally). The experimental results indicate that the two methods are completely consistent and the evaluated result of straightness error by least square method conforms to the least condition.

Key words: straightness error; evaluation; least square method; least envelope zone method

1 引言

GB/T 11336-2004 规定:直线度误差是被测实际直线对其理想直线的变动量,理想直线的位

置应符合最小条件^[1]。根据要求不同,直线度误差分为给定方向上的直线度误差、给定平面内的直线度误差和任意方向上的直线度误差三种。后两种直线度误差在机械产品中常用,如机床导轨的直线度(多用给定平面内的直线度)、回转体轴

心线或孔心线(多用任意方向上的直线度)等。由直线度误差的定义可知,直线度误差评定的关键在于如何根据被测实际直线的提取直线获得满足最小条件的理想直线。目前,直线度误差的评定方法有:最小包容区域法、最小二乘法 and 两端点连线法。其中,两端点连线法作图直观,主要适用于给定平面内的直线度误差的评定,但由于采用两端点连线法获得的理想直线并不能满足最小条件,故其评定误差大,在精密测量特别是仲裁检验中不能采用。最小二乘法是根据残余误差平方和为最小的原理建立理想直线,由于其理论成熟,算法简便,在包括直线度误差在内的形位误差评定中得到广泛的应用。一般认为,采用最小二乘法获得的理想直线也不能满足最小条件,其评定结果与被测直线的真实直线度误差存在一定差异。这一差异实际上就是由评定方法引起的直线度误差与测量结果的系统误差。当被测直线的直线度误差测量精度较高时,用最小二乘法评定直线度误差的评定结果的准确度将受到影响。为此,研究人员研究了多种直线度误差评定方法,如基于遗传算法的直线度误差评定方法^[2]、基于计算机图形技术的直线度误差评定方法^[3]、基于搜索逼近-逐次逼近旋转法的直线度误差评定方法等^[4-9],目的在于建立满足最小条件的理想包容区域,从而获得能真实地反映被测直线的直线度误差大小的评定结果。本文以给定平面内的直线度误差评定为例,详细介绍直线度误差的最小包容区域法的评定原理,并将其与最小二乘评定方法的评定精度做了比较。

2 直线度误差的最小二乘评定方法

$(x_i, y_i), i=1, 2, \dots, n$ 为给定平面内被测实际直线上的采样点坐标值。设由采样点坐标值建立的最小二乘理想直线方程为:

$$y=kx+b, \quad (1)$$

其中 k, b 为待定参数。令残余误差为:

$$\epsilon_i = y_i - (kx_i + b), i=1, 2, \dots, n, \quad (2)$$

最小二乘法的目标函数为: $J(k, b) = \sum_{i=1}^n \epsilon_i^2$, 约束条件是: $J(k, b) \rightarrow \min$ 。要满足约束条件,必须有:

$$\begin{cases} \frac{\partial J}{\partial k} = 0 \\ \frac{\partial J}{\partial b} = 0 \end{cases}, \quad (3)$$

将式(3)简化并求得最小二乘直线的待定参数 k 和 b 为:

$$\begin{cases} k = \frac{n \cdot \sum_{i=1}^n (x_i \cdot y_i) - \sum_{i=1}^n x_i \cdot \sum_{i=1}^n y_i}{n \cdot \sum_{i=1}^n x_i^2 - (\sum_{i=1}^n x_i)^2} \\ b = \frac{\sum_{i=1}^n (x_i \cdot y_i) \cdot \sum_{i=1}^n x_i - \sum_{i=1}^n y_i \cdot (\sum_{i=1}^n x_i)^2}{(\sum_{i=1}^n x_i)^2 - n \cdot \sum_{i=1}^n x_i^2} \end{cases}, \quad (4)$$

根据最小二乘评定方法得被测直线的直线度误差为:

$$\Delta_- = \max\{d_i\} + \max\{d_i'\}, \quad (5)$$

其中 $\max\{d_i\}$ 和 $\max\{d_i'\}$ 分别为最小二乘直线两异侧采样点到最小二乘理想直线的最大距离。

3 直线度误差的最小包容区域评定方法

对于给定平面内直线度误差的评定而言,最小包容区域是指包容实际直线,且具有最小宽度的两平行直线之间的区域。因此,直线度误差的最小包容区域评定方法的关键在于如何根据被测实际直线的提取直线(即采样点)获得满足最小包容区域的理想直线。下面以给定平面内直线度误差的评定为例,介绍基于搜索逼近的最小包容区域评定法的原理与实现方法。

设 $(x_i, y_i), i=1, 2, \dots, n$ 为给定平面内被测实际直线上的采样点坐标值, $y=kx+b$ 为最小二乘法确定的理想直线,其中 k, b 可根据式(4)求得。满足最小包容区域的理想直线可以利用搜索逼近法求得。搜索逼近法的基本思路是:以最小二乘直线为搜索起始直线,以搜索起始直线的斜率增量或减量为搜索步长,在一定的搜索范围内搜索满足最小包容区域的理想直线,并由此理想直线为基准计算直线度误差的评定结果。最小二乘直线与搜索逼近法的理想直线之关系见图 1。

图中 (x_i, y_i) 为被测直线在 $O-xy$ 平面内的采

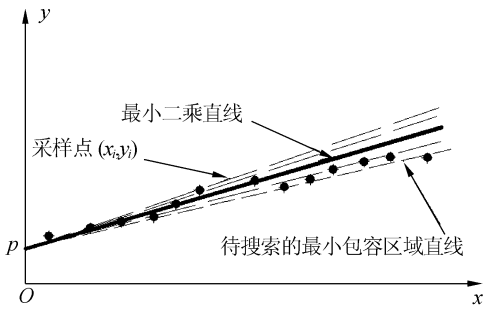


图 1 最小二乘与最小包容区域直线关系图

Fig. 1 Relation of straightness between least square and least envelope zone

样点坐标数据,实线表示最小二乘理想直线,虚线表示待搜索的最小包容区域理想直线。搜索逼近法搜索最小包容区域理想直线的方法与步骤如下:将最小二乘直线 $y = k \cdot x + b$ 进行斜率调整,得待搜索直线的直线方程为:

$$y = k_1 \cdot x + b, \quad (6)$$

其中 $k_1 = k + j \cdot \Delta k$, Δk 为搜索步长, $j = -m, -m+1, \dots, -1, 0, 1, \dots, m-1, m$, m 为正整数,它决定搜索范围的大小。计算对应于 j 的每次搜索的直线度误差值,记为 $|\Delta_-|_j$,当整个设定的搜索区域全部搜索完后,得到满足最小包容区域的直线度误差值为:

$$\Delta_- = \min\{|\Delta_-|_j\}, \quad (7)$$

对应于 $\min\{|\Delta_-|_j\}$ 的直线即为满足最小包容区域的理想直线。搜索逼近法搜索最小包容区域的理想直线的程序框图见图 2。

4 测量实验

为比较最小二乘法与最小包容区域法的直线度误差评定精度,对八种不同对象给定平面内的直线度误差进行测量与评定。每条被测实际直线的长度均为 80 mm,测量采样点采用均匀分布,采样点的间距为 5 mm(即整段直线上的测量采样点为 16 点)。采样点的坐标值在 Global FX777 型三坐标测量机上利用自动扫描(SCAN)功能获得,采样点的坐标数据以 TXT 格式的文件导出。按上述原理利用 MATLAB6.5 软件编程,分别用最小二乘法 and 最小包容区域法评定各被测直线的直线度误差大小。搜索逼近法的初始理想直线用最小二乘直线代替,搜索的范围为最

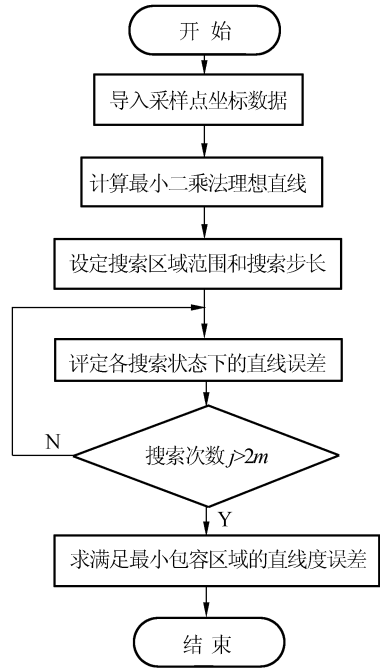


图 2 搜索法的程序框图

Fig. 2 Flow chart of searching

小二乘初始理想直线绕 P 点顺时针和逆时针分别转动 2° ,搜索步长的角度增量或减量为 $0.25'$ 。对应的参数为: $\Delta k \approx 7.27 \times 10^{-5}$, $m = 240$ 。测量实物照片见图 3,评定结果见表 1。从表 1 的比较评定结果可以看出,直线度误差的最小二乘法与最小包容区域法评定精度相同。

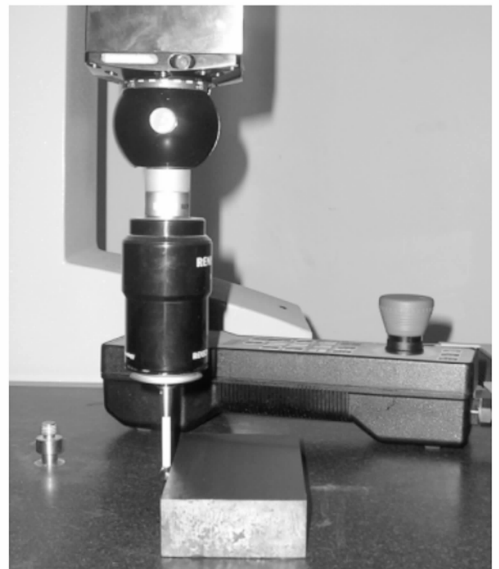


图 3 直线度误差测量实物照片

Fig. 3 Picture of measuring straightness error

表 1 最小二乘法与最小包容区域法评定结果比较表

Tab. 1 Comparison of evaluated results between least square method and least envelope zone method

组别	采样点坐标数据(x 与 y 对应)	结果 1	结果 2
一	x 20.3668,25.3699,30.3414,35.3748,40.3613,45.3355,50.3516,55.3593,60.3633,65.3618,70.3493,75.3474,80.3456,85.3562,90.3540,95.3534	0.0760	0.0760
	y 41.3260,41.2730,41.1855,41.0033,40.8988,40.8197,40.7468,40.6410,40.5309,40.3912,40.2550,40.1489,40.0408,39.9622,39.8411,39.7397		
二	x 56.5288,61.5109,66.5293,71.5218,76.5273,81.5266,86.5257,91.5272,96.5269,101.5227,106.5249,111.5181,116.5205,121.5273,126.5273,131.5235	0.0057	0.0057
	y 17.4280,17.4181,17.4079,17.3926,17.3846,17.3694,17.3624,17.3503,17.3386,17.3305,17.3179,17.3065,17.2950,17.2822,17.2753,17.2607		
三	x 35.6545,40.6444,45.6455,50.6396,55.6405,60.6495,65.6473,70.6431,75.6435,80.6463,85.6450,90.6385,95.6463,100.6487,105.6475,110.6406	0.0074	0.0074
	y 17.3765,17.3635,17.3558,17.3404,17.3326,17.3177,17.3080,17.2933,17.2887,17.2749,17.2656,17.2546,17.2422,17.2318,17.2216,17.2106		
四	x 36.4564,41.4387,46.4442,51.4295,56.4387,61.4336,66.4175,71.4245,76.4301,81.4292,86.4201,91.4001,96.4011,101.4214,106.3915,111.3971	0.0166	0.0166
	y 91.5095,91.6933,91.8673,92.0515,92.2290,92.4123,92.5931,92.7656,92.9469,93.1166,93.3027,93.4715,93.6670,93.8405,94.0215,94.1970		
五	x 55.6191,60.6100,65.6144,70.6104,75.6056,80.6021,85.6082,90.6035,95.5875,100.5835,105.5904,110.5699,115.5854,120.5855,125.5729,130.5607	0.0293	0.0293
	y 12.2003,12.3746,12.5463,12.7078,12.9061,13.0856,13.2720,13.4419,13.6152,13.7963,13.9782,14.1566,14.3297,14.5058,14.6839,14.8700		
六	x 37.5256,42.5382,47.5415,52.5378,57.5339,62.5294,67.5338,72.5328,77.5310,82.5397,87.5317,92.5267,97.5275,102.5211,107.5318,112.5357	0.0160	0.0160
	y 63.3726,63.4182,63.4642,63.5139,63.5626,63.6084,63.6548,63.6917,63.7500,63.7982,63.8472,63.8962,63.9413,63.9885,64.0401,64.0898		
七	x 55.4594,60.4496,65.4571,70.4492,75.4533,80.4519,85.4457,90.4529,95.4525,100.4541,105.4570,110.4318,115.4524,120.4546,125.4559,130.4442	0.0039	0.0039
	y 63.5395,63.5857,63.6339,63.6801,63.7272,63.7757,63.8222,63.8714,63.9189,63.9684,64.0141,64.0604,64.1096,64.1573,64.2077,64.2549		
八	x 44.6308,49.6357,54.6427,59.6362,64.6211,69.6363,74.6255,79.6308,84.6481,89.6141,94.6302,99.6231,104.6158,109.6132,114.6210,119.6087	0.0511	0.0511
	y 40.9328,40.8330,40.7666,40.6557,40.5441,40.4165,40.2886,40.1730,40.0753,39.9832,39.8692,39.7572,39.6718,39.5586,39.4337,39.3198		

注:评定结果 1 为最小二乘法的评定结果,评定结果 2 为最小包容区域法的评定结果;采样点坐标数据与评定结果数据的单位均为 mm

5 结 论

给出了直线度误差评定的最小二乘法与基于搜索逼近-逐次逼近旋转的最小包容区域法的数学模型,并对两种评定方法的算法模型与实现方法进行比较;在三坐标测量机上对给定平面内被

测直线进行采样点坐标数据提取,用最小二乘法和基于搜索逼近-逐次逼近旋转的最小包容区域法对对象直线的直线度误差进行评定,结果表明:最小二乘法与基于搜索逼近-逐次逼近旋转的最小包容区域法的评定结果完全一致,即直线度误差的最小二乘法评定结果符合最小条件。

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